HEATING OF CYLINDRICAL BODIES IN FURNACES FOR ROLLED PRODUCTION

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On the basis of the heat balance equation the solution of the problem of determination of the fuel consumption in heating cylindrical bodies in a furnace for rolled production by an assigned temperature regime with allowance for the temperature distribution over the cross section of the cylindrical article at the current instant of time is considered.

The differential equation of heat conduction in symmetric heating of a cylinder of infinite length is written in the form [1]

$$\frac{\partial T}{\partial \tau} = a \left(\frac{\partial^2 T}{\partial \tau^2} + \frac{1}{x} \frac{\partial T}{\partial x} \right). \tag{1}$$

We consider the problem of heating a cylindrical body in the case of radiative and convective heat transfer on the surface with allowance for the following assumptions [2]: 1) the furnace operates in a stationary mode, i.e., the temperature in any cross section does not change with time; 2) the initial temperature distribution in the billet and the temperature distribution along the furnace length are known; 3) the billets are laid at certain intervals and are heated from all sides; 4) the thermophysical properties of the metal are known; 5) the billets are heated under conditions of radiative-convective heat transfer, and all surfaces are gray.

In the present case we consider an infinite cylinder of radius R with initial temperature T_0. The temperature of the isothermal surface $T(r, \tau)$ of the cylinder at a distance r from the cylinder axis must be determined at the instant of time τ.

The process of heating of the metal can be described by the system of equations:

$$\rho c \frac{\partial T}{\partial \tau} = \lambda \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{r} \right), \quad 0 \le r \le R, \quad 0 < \tau < \tau_{\text{fin}}, \tag{2}$$

with the initial conditions $T(r, 0) = T_0$ and boundary conditions accounting for the heat fluxes absorbed by the surface of the metal due to radiation and convection [3]:

$$\lambda \frac{\partial T(R, \tau)}{\partial r} = \alpha (T_{\text{f}}(\tau) - T(R, \tau)) + \sigma (T_{\text{f}}^4(\tau) - T^4(R, \tau)), \quad \frac{\partial T(0, \tau)}{\partial r}. \tag{3}$$

We assume that the temperature variation along the cylinder length is small and it can be neglected. The temperature distribution of the furnace $T_{\text{f}}(\tau)$ is known for the entire time of heating $0 < \tau \le \tau_{\text{fin}}$.

The bulk-mean temperature of the metal can be calculated via a double integral in polar coordinates:

$$T_{\text{mean}}(\tau) = \frac{1}{F} \iint_D T(r, \tau)\, r dr\, d\varphi = \frac{1}{F} \int_0^{2\pi} d\varphi \int_0^R T(r, \tau)\, r dr = \frac{2\pi}{F} \int_0^R T(r, \tau)\, r dr. \tag{4}$$

It is necessary to obtain the equation for fuel consumption at the instant of time $0 < \tau < \tau_{\text{fin}}$.

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We fix an instant of time τ and some increment of it $\Delta\tau$. On the basis of (4) we can write

$$T_{\text{mean}}(\tau + \Delta\tau) - T_{\text{mean}}(\tau) = \frac{2\pi}{F} \int_0^R (T(r, \tau + \Delta\tau) - T(r, \tau))\, rdr =$$

$$= \frac{2\pi\Delta\tau}{F} \int_0^R \frac{\partial T(r, \theta)}{\partial\tau} rdr, \quad \tau \le \theta \le \tau + \Delta\tau. \tag{5}$$

We consider an integral of the form $\rho c\int_0^R (\partial T(r, \theta)/\partial\tau) rdr$. Taking into account Eq. (2) and boundary conditions (3), we obtain

$$\rho c \int_0^R \frac{\partial T(r, \theta)}{\partial\tau} rdr = \int_0^R \left[\lambda \left(\frac{\partial^2 T(r, \theta)}{\partial r^2} + \frac{1}{r}\frac{\partial T(r, \theta)}{\partial r}\right)\right] rdr =$$

$$= \int_0^R \lambda \frac{\partial^2 T(r, \theta)}{\partial r^2} rdr + \int_0^R \lambda \frac{\partial T(r, \theta)}{\partial r} dr =$$

$$= \lambda \int_0^R \left(\frac{\partial}{\partial r}\left(r\frac{\partial T(r, \theta)}{\partial r}\right) - \frac{\partial T(r, \theta)}{\partial r}\right) dr + \lambda \int_0^R \frac{\partial T(r, \theta)}{\partial r} dr =$$

$$= \lambda r \frac{\partial T(r, \theta)}{\partial r}\bigg|_0^R = \lambda R \frac{\partial T(r, \theta)}{\partial r} = R\, [\alpha\, (T_{\text{f}}(\theta) - T(R, \theta)) + \sigma\, (T_{\text{f}}^4(\theta) - T^4(R, \theta))]. \tag{6}$$

Having substituted (6) into (5), with allowance for the equality $F = \pi R^2$ we have

$$T_{\text{mean}}(\tau + \Delta\tau) - T_{\text{mean}}(\tau) = \frac{2\Delta\tau}{R\rho c}\, [\alpha\, (T_{\text{f}}(\theta) - T(R, \theta)) + \sigma\, (T_{\text{f}}^4(\theta) - T^4(R, \theta))]. \tag{7}$$

We consider the various items of heat consumption that are required to compose the equation of heat balance [1]:

1) for heating of the metal from $T_{\text{mean}}(\tau)$ to $T_{\text{mean}}(\tau + \Delta\tau)$ (useful heat)

$$Q_{\text{use}}(\tau) = \frac{P}{3600} c\, (T_{\text{mean}}(\tau + \Delta\tau) - T_{\text{mean}}(\tau)); \tag{8}$$

2) with stack gases leaving the furnace (for fuel furnaces)

$$Q_{\text{leav}}(\tau) = B(\tau)\, V_{\text{s}} c_{\text{s}} T_{\text{leav}} \Delta\tau; \tag{9}$$

3) due to losses through the furnace lining

$$Q_{\text{lin}}(\tau) = \frac{T_{\text{f}}(\tau) - T_{\text{env}}}{S_1/\lambda_1 + S_2/\lambda_2 + 1/\alpha_{\text{med}}} F_{\text{w}} \Delta\tau; \tag{10}$$

4) by radiation through open windows and doors of the furnace

$$Q_{\text{h}}(\tau) = c_0\, (T_{\text{f}}(\tau)/100)^4\, \Phi\psi F_{\text{wind}} \Delta\tau; \tag{11}$$

5) for heating of transporting devices (trays, conveyers)

$$Q_{\text{tr}}(\tau) = M_{\text{tr}} c_{\text{tr}}\, (T_{\text{tr}}(\tau + \Delta\tau) - T_{\text{tr}}(\tau)),$$

and assuming that the change in temperature in heating the transporting devices corresponds to the change in the furnace temperature:

$$Q_{tr}(\tau) = M_{tr}c_{tr}(T_f(\tau + \Delta\tau) - T_f(\tau)) ; \tag{12}$$

6) due to losses through the metal rods and inserts in the furnace lining, called thermal short circuits:

$$Q_{th.sh.c}(\tau) \approx Q_{lin}(\tau) ; \tag{13}$$

7) unaccounted losses

$$Q_{unacc}(\tau) = 0.1\,(Q_{lin}(\tau) + Q_h(\tau) + Q_{tr}(\tau) + Q_{th.sh.c}(\tau)) . \tag{14}$$

Thus, the total heat consumption in a fuel furnace at the instant of time τ is determined as

$$\Sigma\, Q_{cons}(\tau) = Q_{use}(\tau) + Q_{leav}(\tau) + Q_{lin}(\tau) + Q_h(\tau) + Q_{tr}(\tau) + Q_{th.sh.c}(\tau) + Q_{unacc}(\tau) . \tag{15}$$

Heat input consists of the following items:

1) due to fuel combustion (the heat of the chemical reactions of combustion)

$$Q_{ch.h.fuel}(\tau) = B(\tau)\, Q_{low}^{work} \Delta\tau ; \tag{16}$$

2) in heating of air supplied for fuel combustion (the physical heat of the air)

$$Q_{ph.h.a}(\tau) = B(\tau)\, V_a c_a T_a \Delta\tau , \tag{17}$$

3) in heating of the fuel (the physical heat of the fuel)

$$Q_{ph.h.fuel}(\tau) = B(\tau)\, c_{fuel} T_{fuel} \Delta\tau . \tag{18}$$

Thus, the total heat input in a fuel furnace at the instant of time τ is determined as

$$\Sigma\, Q_{in}(\tau) = Q_{ch.h.fuel}(\tau) + Q_{ph.h.a}(\tau) + Q_{ph.h.fuel}(\tau) . \tag{19}$$

According to the law of energy conservation the total consumption of heat should be compensated by the total input of heat to the furnace. Therefore, the equation of heat balance has the form

$$\Sigma\, Q_{cons}(\tau) = \Sigma\, Q_{in}(\tau) . \tag{20}$$

Substituting (15), (19) in (20) and taking into account Eqs. (8)-(14), (16)-(18), we have

$$\frac{P}{3600} c\,(T_{mean}(\tau + \Delta\tau) - T_{mean}(\tau)) + B(\tau)\, V_s c_s T_{leav} \Delta\tau +$$

$$+ 2.2 \frac{T_f(\tau) - T_{sur}}{S_1/\lambda_1 + S_2/\lambda_2 + 1/\alpha_{med}} F_w \Delta\tau + 1.1\, c_0 (T_f(\tau)/100)^4\, \Phi\psi F \Delta\tau +$$

$$+ 1.1\, M_{tr} c_{tr} (T_f(\tau + \Delta\tau) - T_f(\tau)) = B(\tau)\, Q_{low}^{work} \Delta\tau + B(\tau)\, V_a c_a T_a \Delta\tau + B(\tau)\, c_{fuel} T_{fuel} \Delta\tau . \tag{21}$$

Substituting (7) in (21), we combine similar terms, divide by $\Delta\tau$, and pass to the limit as $\Delta\tau \to 0$. Assuming that

$$\lim_{\Delta\tau \to 0} \frac{T_f(\tau + \Delta\tau) - T_f(\tau)}{\Delta\tau} = \frac{dT_f(\tau)}{d\tau} \quad \text{and} \quad \lim_{\Delta\tau \to 0} T(r, \theta) = T(r, \tau) , \quad \tau \le \theta \le \tau + \Delta\tau ,$$

we finally have

$$\frac{dT_f(\tau)}{d\tau} = A_1 B(\tau) - A_2 T_f(\tau) + A_3 T(R, \tau) - A_4 T_f^4(\tau) + A_5 T^4(R, \tau) + A_6 T_{sur}, \tag{22}$$

where

$$A_1 = \frac{Q_{low}^{work} + V_a c_a T_a + c_{fuel} T_{fuel} - V_s c_s T_{dep}}{1.1 M_{tr} c_{tr}};$$

$$A_2 = \frac{1}{M_{tr} c_{tr}} \left(\frac{P\alpha}{1980 R\rho} + \frac{2F_w}{S_1/\lambda_1 + S_2/\lambda_2 + 1/\alpha_{med}} \right);$$

$$A_3 = \frac{P\alpha}{1800 R\rho}; \quad A_4 = \frac{1}{M_{tr} c_{tr}} \left(\frac{P\sigma}{1980 R\rho} + \frac{c_0 \Phi \psi F}{10^8} \right);$$

$$A_5 = \frac{P\sigma}{1800 R\rho}; \quad A_6 = \frac{F_w}{S_1/\lambda_1 + S_2/\lambda_2 + 1/\alpha_{med}}.$$

Equation (22) allows one to determine the value of the fuel consumption at the instant of time $0 < \tau \le \tau_{fin}$ that is necessary to maintain the required temperature mode in the furnace. In contrast to the approach described in [1], the technique suggested allows for the temperature distribution over the cross section of the cylindrical article at the current instant of time rather than the initial and final temperatures of the metal.

Thus, a technique for determining fuel consumption during the entire period of heating is developed on the basis of construction of the equation of heat balance for a heating furnace for rolled production. The results of the paper may be used to identify actual processes of heating cylindrical bodies in a furnace, e.g., the data of experimental study of heating a metal in [3, 4], with the purpose of subsequent optimization of these processes.

NOTATION

T, temperature, K; τ, current time, h; a, coefficient of thermal diffusivity of the material, m^2/h, $a = \lambda/(\rho c)$; λ, thermal conductivity, $J/(m \cdot h \cdot K)$; c, heat capacity of the metal, $J/(kg \cdot K)$; ρ, density of the material, kg/m^3; r, current distance from the cylinder axis, m; R, cylinder radius, m; τ_{fin}, time of heating process termination, h; T_0, initial uniform distribution of the temperature in the cylinder, K; α, coefficient of convective heat transfer, $W/(m^2 \cdot K)$; σ, coefficient of radiative heat transfer, $W/(m^2 \cdot K^4)$; $T_f(\tau)$, temperature in the furnace at the instant of time τ; $T_{mean}(\tau)$, $T_{mean}(\tau + \Delta\tau)$, bulk-mean temperatures the instants of time τ and $\tau + \Delta\tau$; F, cylinder cross section, m^2; P, furnace efficiency, kg/h; $B(\tau)$, fuel consumption, kg/sec, m^3/sec; V_s, volume of the combustion products formed in combustion of 1 kg or 1 m^3 of fuel, m^3/kg, m^3/m^3; c_s, specific heat of the combustion products, $J/(m^3 \cdot K)$; T_{leav}, temperature of the departing stack gases (taken according to the temperature mode of the furnace), K; T_{sur}, temperature of the surrounding air, K; $S_1/\lambda_1, S_2/\lambda_2$, thermal resistances (ratios of the thicknesses of the lining layers to their coefficients of thermal conductivity), $m^2 \cdot K/W$; α_{med}, coefficient of heat transfer from the outer surface of the furnace walls to the surrounding medium, $W/(m^2 \cdot K)$; F_w, area of the outer surface of the furnace lining, m^2; $c_0 = 5.7\ W/(m^2 \cdot K^4)$, coefficient of emissivity; Φ, coefficient of diaphragming (determined from the plot of [1], p. 49); ψ, relative time of opening a window or door (if a window is open for 30 min in 1 h, $\psi = 0.5$); F_{wind}, area of a window or door, m^2; M_{tr}, c_{tr}, mass (kg) and mean specific heat ($J/(kg \cdot K)$) of the transporting devices present in the furnace in the time $\Delta\tau$, respectively; $T_{tr}(\tau)$, $T_{tr}(\tau + \Delta\tau)$, temperatures of the transporting devices at the instants of time τ and $\tau + \Delta\tau$, respectively, K; Q_{low}^{work}, lowest heat of fuel combustion, J/kg, J/m^3; V_a, volume of air necessary for combustion of 1 kg or 1 m^3 of fuel (with account for the required excess of air), m^3/kg, m^3/m^3; c_a, mean specific heat of the air, $J/(m^3 \cdot K)$; T_a, temperature of heating the air, K; c_{fuel}, mean specific heat of the fuel, $J/(kg \cdot K)$, $J/(m^3 \cdot K)$; T_{fuel}, temperature of heating the fuel, K. Subscripts and superscripts: fin, final; f, furnace; mean, mean; s, smoke; leav, leaving; sur, surrounding; med, medium; w, wall; wind, window; tr, transporting; low, lowest; work, working; a, air; fuel, fuel; use, useful; lin, lining; h, hole; th.sh.c, thermal short circuit; unacc, unaccounted; cons, consumption; ch.h.fuel, chemical heat of the fuel; ph.h.a, physical heat of the air; ph.h.fuel, physical heat of the fuel; in, input.

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